

Příklad 1: Integrujte pomocí V2

a) $\int \left(x^5 - \sqrt[4]{x} + \frac{1}{x^3} \right) dx = \frac{x^6}{6} - \frac{4}{5} \sqrt[4]{x^5} - \frac{1}{2x^2} + C$

b) $\int \frac{(x+3)^2}{x^2} dx = x + 6 \ln|x| - \frac{9}{x} + C$

c) $\int \frac{\sqrt{x} - 2 \cdot \sqrt[3]{x}}{x} dx = 2\sqrt{x} - 6 \cdot \sqrt[3]{x} + C$

d) $\int x \left(\sqrt{x} + \frac{1}{x^4} \right) dx = \frac{2}{5} \sqrt{x^5} - \frac{1}{2x^2} + C$

Příklad 2: Integrujte pomocí V8

a) $\int \frac{4}{3-x} dx = -4 \ln|3-x| + C$

b) $\int \frac{x}{x^2 + 3} dx = \frac{1}{2} \ln|x^2 + 3| + C$

c) $\int \frac{5x^2}{x^3 + 1} dx = \frac{5}{3} \ln|x^3 + 1| + C$

d) $\int 3 \cot g x dx = 3 \ln|\sin x| + C$

Příklad 3: Integrujte pomocí V9

a) $\int \sin(3x + 4) dx = -\frac{1}{3} \cos(3x + 4) + C$

b) $\int \sqrt[3]{5x-7} dx = \frac{3}{20} \cdot \sqrt[3]{(5x-7)^4} + C$

c) $\int e^{-x+2} dx = -e^{-x+2} + C$

d) $\int \frac{1}{(4x-1)^3} dx = -\frac{1}{8} \cdot \frac{1}{(4x-1)^2} + C$

Příklad 4: Integrujte pomocí V12,V13

a) $\int \frac{3}{x^2 + 5} dx = \frac{3}{\sqrt{5}} \operatorname{arctg} \frac{x}{\sqrt{5}} + C$

b) $\int \frac{2}{2 - x^2} dx = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{2} + x}{\sqrt{2} - x} \right| + C$

c) $\int \frac{1}{x^2 + 3x + 3} dx = \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2x + 3}{\sqrt{3}} + C$

d) $\int \frac{1}{x^2 + 6x + 5} dx = -\frac{1}{4} \ln \left| \frac{5 + x}{-1 - x} \right| + C$

Příklad 5: Integrujte pomocí V14,V15

a) $\int \frac{1}{\sqrt{x^2 - 5}} dx = \ln \left| x + \sqrt{x^2 - 5} \right| + C$

b) $\int \frac{1}{\sqrt{5 - x^2}} dx = \arcsin \frac{x}{\sqrt{5}} + C$

c) $\int \frac{2}{\sqrt{x^2 - 4x + 1}} dx = 2 \ln \left| x - 2 + \sqrt{x^2 - 4x + 1} \right| + C$

d) $\int \frac{1}{\sqrt{-x^2 - 2x}} dx = \arcsin(x + 1) + C$

Příklad 6: Pomocí vzorců a s využitím úprav integrandu vypočtěte integrály

a) $\int 2 \operatorname{tg} x dx = -2 \ln |\cos x| + C$

b) $\int \frac{1}{\sqrt{x^2 + 6x + 10}} dx = \ln \left| x + 3 + \sqrt{x^2 + 6x + 10} \right| + C$

c) $\int \frac{2 \cos x}{4 + \sin x} dx = 2 \ln |4 + \sin x| + C$

d) $\int \frac{4 - \cos 3x}{2} dx = 2x - \frac{1}{6} \sin 3x + C$

$$\mathbf{e)} \int \frac{x^3 + 2x}{\sqrt{x}} dx = \frac{2}{7} \sqrt{x^7} + \frac{4}{3} \sqrt{x^3} + C$$

$$\mathbf{f)} \int \frac{2}{x^2 + 4x + 4} dx = \frac{-2}{x+2} + C$$

$$\mathbf{g)} \int (x^3 + 3x)(x - 2) dx = \frac{1}{5}x^5 - \frac{1}{2}x^4 + x^3 - 3x^2 + C$$

$$\mathbf{h)} \int \cot^2 x dx = -\cot x - x + C$$

$$\mathbf{i)} \int \frac{6}{x^2 + 4x - 5} dx = -\ln \left| \frac{5+x}{1-x} \right| + C$$

$$\mathbf{j)} \int \frac{10}{\cos^2(2x)} dx = 5\tg(2x) + C$$

$$\mathbf{k)} \int x^2 \left(\frac{1}{4x} - 4x + 4\sqrt{x} \right) dx = \frac{1}{8}x^2 - x^4 + \frac{8}{7}\sqrt{x^7} + C$$

Příklad 7: Integrujte racionální funkce (neryze lomené rozložte)

$$\mathbf{a)} \int \frac{3}{2+x^2} dx = \frac{3}{\sqrt{2}} \arctg \frac{x}{\sqrt{2}} + C$$

$$\mathbf{b)} \int \frac{x}{x-3} dx = x + 3 \ln|x-3| + C$$

$$\mathbf{c)} \int \frac{1}{2x-3} dx = \frac{1}{2} \ln|2x-3| + C$$

$$\mathbf{d)} \int \frac{x^2}{x^2+3} dx = x - \frac{3}{\sqrt{3}} \arctg \frac{x}{\sqrt{3}} + C$$

$$\mathbf{e)} \int \frac{x^3}{x^2+3} dx = \frac{x^2}{2} - \frac{3}{2} \ln|x^2+3| + C$$

$$\mathbf{f)} \int \frac{4}{(x-2)^3} dx = \frac{-2}{(x-2)^2} + C$$

$$\mathbf{g)} \int \frac{x^3-5}{x^2} dx = \frac{x^2}{2} + \frac{5}{x} + C$$

$$\mathbf{h}) \int \frac{x^4}{x^2 + 1} dx = \frac{x^3}{3} - x + \arctg x + C$$

$$\mathbf{i}) \int \frac{x^2}{x-1} dx = \frac{x^2}{2} + x + \ln|x-1| + C$$

$$\mathbf{j}) \int \frac{x^2 + 2x}{x^2 - 1} dx = x + \ln|x^2 - 1| - \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$$

$$\mathbf{k}) \int \frac{1}{x^2 - 4x} dx = -\frac{1}{4} \ln \left| \frac{x}{4-x} \right| + C$$

$$\mathbf{l}) \int \frac{2x}{x+5} dx = 2x - 10 \ln|x+5| + C$$